# Inflation-linked Investment Objectives with Liquid Asset Portfolios

Many investors, both institutional as well as retail, have investment objective functions formulated in terms of exceeding a target relative to inflation. Building a portfolio that maximises the probability of meeting such objectives needs to take a different tack than pure asset-only portfolios. In this paper we consider the constrained set of liquid investments only with a view to selecting portfolios whose return objectives are inflation-relative. The risk criteria are also expressed in these terms, leading to a trade-off in risk/return space that is different from traditional asset-only portfolio selection.

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"Inflation Plus" investment targets suit many types of investor, and are usually derived from an underlying liability structure or spending pattern.

## Inflation-linked investment objectives

For investors whose liabilities or spending objectives are linked to inflation the step to linking their investment objectives to inflation as well is a fairly natural one. Whether this makes sense in all cases is another matter as other considerations also play a role in the investment strategy and the risk assessment, but for many investors an "Inflation Plus" investment objective does suit their requirements. For one, it is easily intelligible and it also reifies the link between the inflation-linked components of the liabilities and the investment strategy. Secondly it also provides a relatively easy way to measure the performance of the investment portfolio, and thirdly it allows the investment policy to be set and reviewed without having to revisit the liabilities in every case.

Although we would argue that ideally the liabilities need to be modelled explicitly to optimise an investment strategy with acceptable levels of risk, in many cases the formulation of the "Inflation Plus" investment objective does suffice. In this paper we examine the construction of an investment portfolio that seeks to meet a pre-determined "Inflation Plus" return target,[1] using the Australian market as an example.

This kind of target has a number of broad applications. The first case would be a retiree who uses his defined contribution savings in order to provide for old-age income. This spending objective is specified as a percentage of the nominal sum at retirement, but indexed with inflation over time so that real spending is maintained. The second case would be an endowment that has a spending goal expressed as a percentage of the available funds with preservation of real capital being crucial to ensure perpetuity in its operations.

## Focus on liquid assets

By taking the "Inflation Plus" target as a given we do not investigate the underlying liabilities or spending patterns that led to the formulation of this goal. We assume that the investor is essentially creating a perpetuity that allows for indefinite spending at the set level, with an acceptable risk level set at various future points in time. Most importantly though, this analysis restricts itself to investments in liquid asset classes only.

An institutional investor with a long enough investment horizon and suitable liquidity requirements may be able to avail itself of liquidity premia in additional asset classes, and also may be able to hedge its inflation and interest rate exposure directly through swaps. This would lead to a completely different portfolio than the one presented here.[2] The decision to restrict ourselves to liquid assets only allows to us to present a solution that is widely applicable without loss of generality.

[1] In a separate forthcoming paper we will look at the determination of an "Inflation Plus" target by analysing the liabilities of an institutional investor.

[2] A companion paper to this one will be forthcoming which expands the investable universe to illiquid assets and explicit inflation and interest rate hedges.

Another important aspect to note is that in building a portfolio that has an "Inflation Plus" target we are not solely seeking to hedge inflation, but to add returns over and above inflation. This is an important distinction, because if the hedge were the sole purpose, the portfolio selection and risk criteria would be different.

## Asset classes and their inflation-linked behaviour

As we are using an Australian investor as an example in this paper, the analysis must start with Australian inflation. The publication of inflation data in Australia is fairly unique in that it is reported only on a quarterly basis. This is not a major impediment in the context of this paper as we seek to investigate the long-term relationships between inflation and various asset categories. Capital market noise in monthly data leads to distortions in the observation of what should be longer-term characteristics. All data used in this analysis is therefore based on quarterly observations, spanning the period from the quarter ending 30 June 1991 to the quarter ending 31 March 2012 for a total of 84 observations, with all data in AUD unless otherwise specified.

To start the analysis we included a large number of potential asset classes, ranging from Australian dollar cash, through nominal bonds, inflation-linked bonds, domestic and international equities, commodities, listed real estate to volatility as an asset class. We chose a wide range of liquid assets in order to explore a sufficiently broad gamut of potential inflation-hedging possibilities as well as return-enhancing ones.

The relationship between inflation and asset returns has been documented in the literature,[3] including numerous studies that by far predate the inception of the inflation-linked bond market, such as Fama and Schwert 1977. Typically equities as an asset class have been poor hedges of inflation, sometimes even with statistically significant negative correlations to inflation as documented by for instance Froot 1995 and Hoevenaars, et al. 2008. Various explanations have been proferred to explain this negative correlation as in Stulz 1986 and Asikoglu and Ercan 1992, which also include surveys of the literature. Our findings for Australian inflation are largely consistent with these results, showing significantly negative correlations of inflation with the overall Australian stock market, and on a sector level only two positive weakly significant correlations in the Basic Materials and Oil and Gas sectors as shown in Table 1.

[3] For example, see Markowitz (1991).

Table 1

| Asset Class | Correlation with Australian CPI | t-value | p-value |
|---|---|---|---|
| GSCI Energy | 0.41 | 4.11 | 0.0% |
| GDP Australia | –0.40 | –3.97 | 0.0% |
| GSCI Precious Metals | 0.32 | 3.02 | 0.5% |
| GSCI Agricultural | 0.30 | 2.89 | 0.7% |
| AU Consumer Goods | –0.29 | –2.71 | 1.1% |
| GSCI Industrial Metals | 0.24 | 2.22 | 3.5% |
| Australian Bonds | –0.23 | –2.17 | 3.9% |
| AU Industrials | –0.22 | –2.08 | 4.7% |
| AU Basic Materials | 0.21 | 1.94 | 6.2% |
| AU Oil and Gas | 0.21 | 1.91 | 6.5% |
| World Equities | –0.17 | –1.56 | 11.9% |
| 3-month Interest Rate Australia | 0.17 | 1.52 | 12.6% |
| AU Utilities | –0.16 | –1.42 | 14.5% |
| Emerging Markets Equities | –0.14 | –1.25 | 18.2% |
| Australian ILBs | –0.13 | –1.20 | 19.3% |
| GSCI Livestock | –0.12 | –1.14 | 20.8% |
| AU Real Estate | –0.11 | –0.99 | 24.2% |
| AU Health Care | –0.08 | –0.72 | 30.5% |
| AU REITS | –0.08 | –0.72 | 30.7% |
| AU Consumer Services | –0.08 | –0.69 | 31.3% |
| Volatility | –0.08 | –0.68 | 31.5% |
| Global Bonds | –0.06 | –0.53 | 34.5% |
| 10 year Bond Yield Australia | 0.03 | 0.23 | 38.7% |
| Global Property | –0.01 | –0.05 | 39.7% |

Statistically significant correlations are highlighted at the top of the table.

Nominal bonds showed a significant negative correlation with inflation, as might be expected. After all, interest rates and bond yields are expected to rise with rising inflation, depressing total returns. More surprising however might be the fact the Australian inflation-linked bonds showed a statistically insignificant negative correlation with inflation. The possibilities offered by inflation-linked bonds were explored well before their introduction by *Bodie, Inflation, Index-Linked Bonds and Asset Allocation 1988* and shortly after the introduction of TIPS in the United States by *McFall Lamm, Jr. 1998*.

The reason for this lack of correlation is the mark-to-market noise introduced into the total returns by movements in both real and nominal yields. If the bonds were held to maturity the real yield over inflation would be realised, but other factors impinging on the inflation-linked bond market make it a poor inflation hedge if one has to mark one's portfolios to market.

A good overview of the factors driving inflation-linked bonds can be found in *Campbell, Shiller and Viceira 2009*. As we are considering liquid mark-to-market portfolios here, we do not explore the hold-to-maturity strategy in detail, but instead focus on inflation-linked bonds as a liquid investment.

Commodities do seem to possess inflation-hedging characteristics, with significant correlations for the Energy, Precious Metals, Agricultural and Industrials sub-indices of the S&P GSCI commodities index series. The highest correlation at 0.41 is recorded by the Energy sub-index, which is mildly remarkable as the Australian CPI basket does not contain an explicit energy component. Nonetheless the indirect impact of energy prices seems to be sufficiently large to result in a relatively high correlation. The inflation-hedging properties of commodities have also been well documented in the literature, for instance in *Bodie, Commodity Futures as a Hedge Against Inflation 1983*, *Greer 1978* and *Amenc, Martellini and Ziemann 2009*.

Listed property securities turn out not to have any great inflation-hedging characteristics, which is consistent with the literature; for instance *Stevenson 2001* provides a literature survey and updates results, although *Amenc, Martellini and Ziemann 2009* do find an inflation-hedging role for listed real estate. We also included volatility as an asset class as a possible hedge against tail risk as described in *Baars, Kocourek and van der Lende 2012*.

We should not lose sight however of the fact that we are not seeking to hedge inflation but to create a return stream that outperforms inflation with a certain amount. While the inflation-relative characteristics are important, the real expected returns of the assets need to be taken into account a swell.

## Shrinking the investable universe

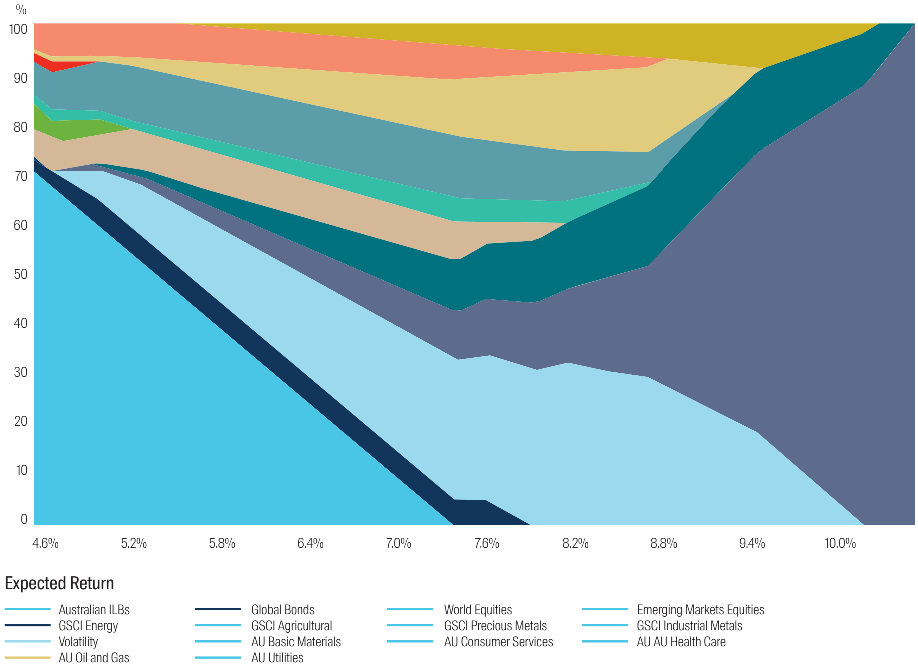
Having established a range of potential asset classes to invest in, we now need to find the combination of assets that best meets the objectives within reasonable risk limits. In an ideal world we would have an infinite amount of computational capacity and would be able to calculate characteristics for all asset combinations. In practice however we need to shrink the possible number of portfolios to be checked down to a smaller number of candidates. This can be done in a number of ways, from naïve application of modern portfolio theory to using risk-weighted baskets, minimum variance portfolios, principal components-weighted portfolios, copula-based approaches and others.[4] See also Lee 2011 for an overview and our forthcoming paper on this topic.

Table 2

| Asset Classes | Expected Return | Expected Volatility |
|---|---|---|
| 3-month Interest Rate Australia | 3.5% | 1.1% |
| 10 year Bond Yield Australia | 3.9% | 1.4% |
| GDP Australia | 3.0% | 1.2% |
| Price Inflation Australia | 2.5% | 1.3% |
| Australian Bonds | 4.0% | 5.3% |
| Australian ILBs | 4.0% | 5.0% |
| Global Bonds | 3.0% | 13.4% |
| World Equities | 8.5% | 17.6% |
| Emerging Markets Equities | 10.5% | 25.7% |
| Global Property | 8.0% | 20.2% |
| GSCI Energy | 7.8% | 28.3% |
| GSCI Agricultural | 4.2% | 18.8% |
| GSCI Livestock | 4.2% | 18.9% |
| GSCI Precious Metals | 3.6% | 12.9% |
| GSCI Industrial Metals | 5.7% | 23.8% |
| Volatility | –1.8% | 49.9% |
| AU Basic Materials | 8.5% | 21.5% |
| AU Consumer Goods | 7.97% | 30.2% |
| AU Consumer Services | 8.0% | 23.0% |
| AU Health Care | 8.0% | 16.8% |
| AU Industrials | 8.2% | 23.0% |
| AU Oil and Gas | 8.5% | 21.4% |
| AU REITS | 8.0% | 21.4% |
| AU Real Estate | 8.0% | 21.8% |
| AU Utilities | 8.0% | 22.6% |

[4] For investors who can invest in illiquid assets and buy explicit inflation protection through swaps, this becomes much more interesting as one can cleanly separate the hedging portfolio from the alpha-generating portfolio. We will explore this topic in a forthcoming paper that includes illiquid assets and the use of swaps.

Figure 1: Mean-Expected Conditional Value at Risk Efficient Frontier Weights




For the sake of brevity we confine ourselves here to an inflation-relative CVaR optimisation,using the output of our stochastic Long-Term Expected Return Model (LTARM)[5] as inputs, together with the covariances of the assets and inflation which we show in Table 2. This results in an inflation-relative efficient frontier in CVaR space which we will use as our range of candidate strategies for further investigation. Figure 1 shows the weights (as vertical slices in the chart) of the assets along this efficient frontier, ranging from the most conservative portfolios on the left to the most aggressive on the right. The horizontal axis shows the nomina lexpected return of each portfolio. The actual optimisation and portfolio selection is based on a stochastic approach using our vector-autoregressive (VAR) ALM model, which also takes autocorrelations into account.

[5] Further information available upon request.

Any forecasts in the charts/graphs represent hypothetical numbers and are purely for illustrative purposes only. The numbers do not represent actual or future performance.

Choosing an "Acceptable Risk" level is case-dependent but has an important impact on the results.

## Study parameters

We distinguish two cases, one with an investment objective of inflation plus 3.5% per annum, the other inflation plus 4.5% per annum. The objective is to maximise real assets over time given annual real spending of 3.5% and 4.5% of initial capital respectively. In both cases we use an evaluation horizon of 25 years, which is more than long enough for the simulations in the VAR model to stabilise and also roughly matches a realistic retirement period assuming retirement at age 65.

| Investment Objective | Evaluation Time Horizon (years) | Worst Case Probability | Acceptable Risk Level (% age of Real Assets) |
|---|---|---|---|
| Inflation + 3.5% | 25 | 5% | 25.0% |
| Inflation + 4.5% | 25 | 10% | 0.0% |

In deciding on what constitutes an "Acceptable Risk Level" in either case we use remaining real assets as a criterion, but with different levels of confidence. For the 3.5% spending case we demand that the real assets in the 5% worst case outcomes are 25% or more of initial assets, whereas for the 4.5% spending case we demand that in the 10% worst case outcomes real assets are greater than zero. Additionally we would like to see real assets in the expected outcome remain at least constant.

Using our proprietary ALM model we ran 2,500 simulations of future time paths, fully incorporating the covariances, autocorrelations, stochastic yield curves and discount rates. The spending in any given year was kept constant in real terms.

## Optimising real assets over time

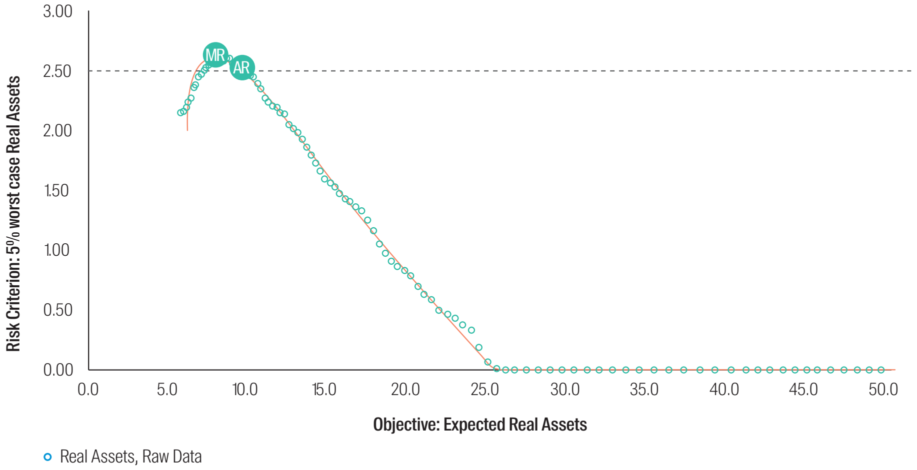
The actual optimisation and portfolio selection takes place in the real asset space, rather than in the more traditional space of return and volatility. As our long-term objectives are stated in terms of real assets and spending, this evaluation space makes more sense. Based upon the parameters as described in the previous section, we present the set of candidate portfolios in a number of graphs that illustrate the risk/return tradeoff we are making.

Figure 2 shows the case for annual real spending of 3.5% of initial capital, equalling an investment objective of CPI+3.5%. The blue circles show the raw simulation outcomes from our model and the solid black line represents the fitted trend. The horizontal axis shows the expected real assets in year 25, with 10.0 being equal to constant real assets over time. The vertical axis shows the 5% worst case outcome for real assets; if maximising the real assets in the worst case were the only consideration, then the "MR" or Minimum Risk portfolio at the top of the graph would be the recommended choice.

However as we allow an Acceptable Risk Level of 25% of initial real assets, the "AR" or Acceptable Risk portfolio is a bit further to the right in the chart.[6] Figure 3 is a zoomed-in version of the relevant area. This Acceptable Risk portfolio allows for a better outcome in the expected case while still meeting the risk criterion.

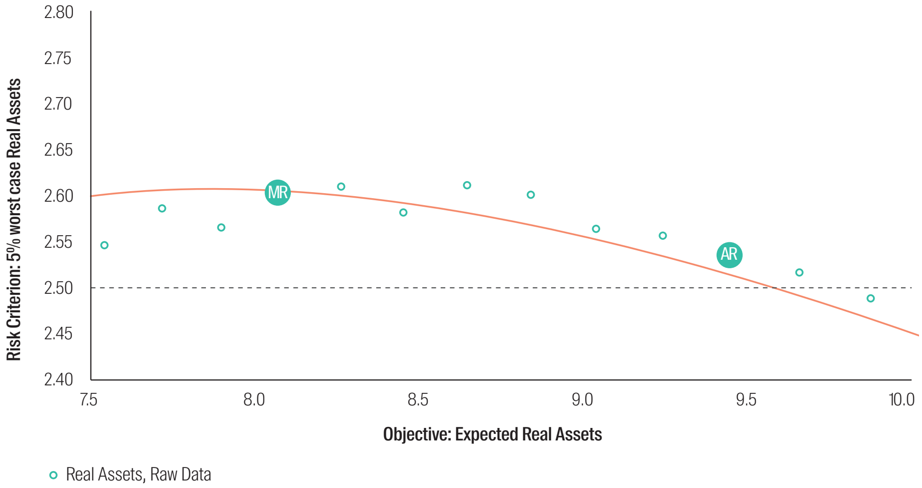
6 In order to meet the requirement we demand that both the raw simulation outcome as well as the fitted trend have to be greater than or equal to the Acceptable Risk Level.

Figure 2: Investment Strategy Risk Analysis, CPI + 3.5%




Source: First Sentier Investors

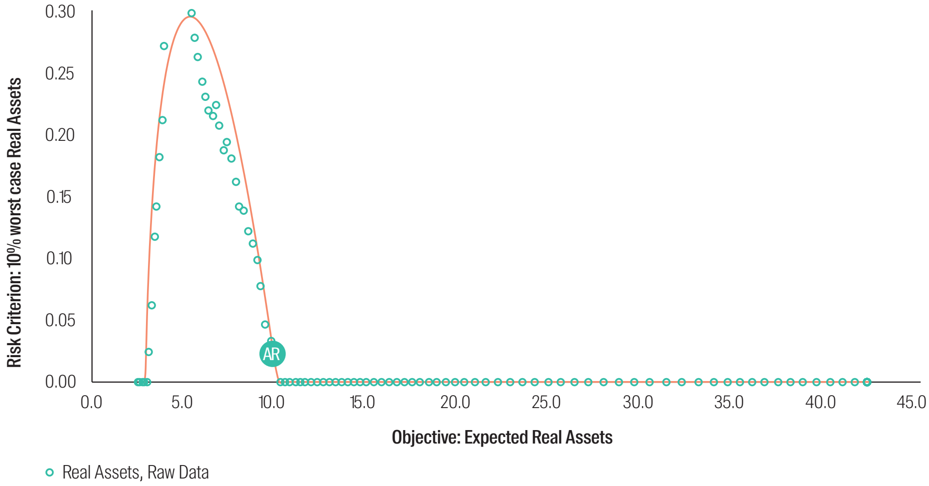
Figure 3: Investment Strategy Risk Analysis, CPI + 3.5%




Source: First Sentier Investors

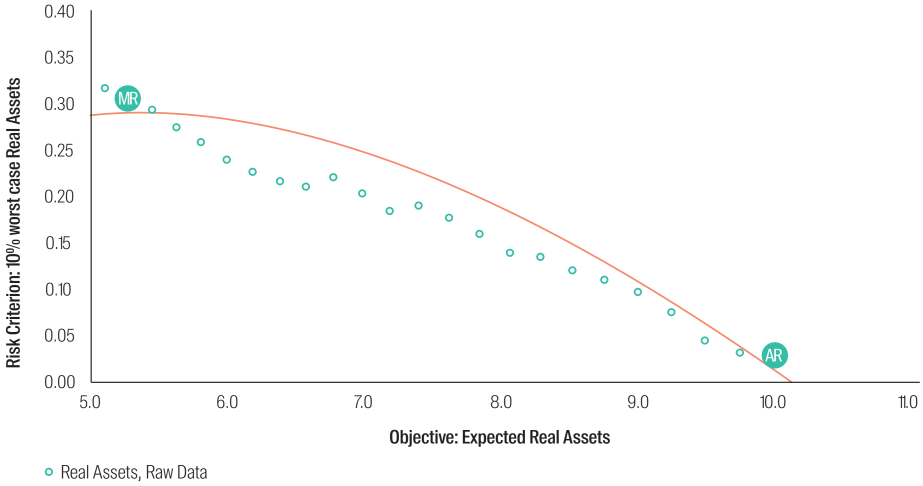
Analogously we show the CPI+4.5% target (with annual real withdrawals equalling 4.5% of initial capital) in Figure 4, where the horizontal axis is again the expected amount of real assets, but the vertical axis is now the 10% worst case outcome. We had set the Acceptable Risk Level at 0% after 25 years in this worst case, and Figure 5 shows the zoomed-in version of the chart.

Figure 4: Investment Strategy Risk Analysis, CPI + 4.5%




Source: First Sentier Investors

Figure 5: Investment Strategy Risk Analysis, CPI + 4.5%




Source: First Sentier Investors

Any forecasts in the charts/graphs represent hypothetical numbers and are purely for illustrative purposes only. The numbers do not represent actual or future performance.

These charts form the essence of the risk/return trade-off. As we have not modelled any liabilities beyond a simple spending pattern the trade-off is relatively straightforward. In cases where we do model the full liability structure the outcome can be complex, and indeed lead to a formulation of an "Inflation Plus" target rather than treating it as an ex-ante given as we have done in this analysis.

## Characteristics of the selected portfolios

The actual composition of the two Acceptable Risk portfolios for the cases of 3.5% and 4.5% real return targets is shown in Table 3. Unsurprisingly the allocation to risky assets is higher in the case where the portfolio has a higher return target, and indeed the alpha-generating aspects of the portfolio start to dominate relative to the inflation-hedging components.

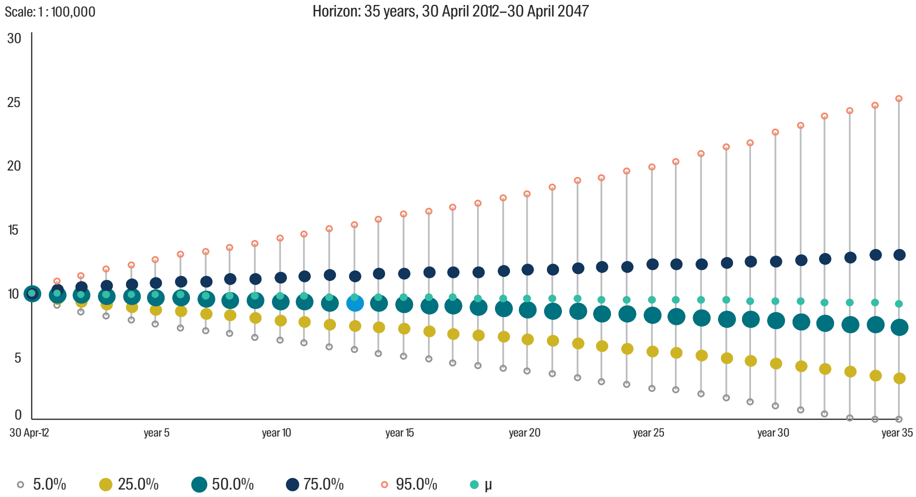
Table 3

| | Acceptable Risk PortfolioWeights | |
|---|---|---|
| | CPI+3.5% | CPI+4.5% |
| Objective | 0.94 | 1.00 |
| Risk Criterion | 0.25 | 0.00 |
| Australian Bonds | 0.0% | 0.0% |
| Australian ILBs | 35.6% | 8.7% |
| Global Bonds | 5.0% | 5.0% |
| World Equities | 18.2% | 29.6% |
| Emerging Markets Equities | 0.0% | 1.5% |
| Global Property | 0.0% | 0.0% |
| GSCI Energy | 1.9% | 5.3% |
| GSCI Agricultural | 0.0% | 0.0% |
| GSCI Livestock | 0.0% | 0.0% |
| GSCI Precious Metals | 7.5% | 7.5% |
| GSCI Industrial Metals | 0.0% | 0.0% |
| Volatility | 3.3% | 4.3% |
| AU Basic Materials | 13.8% | 14.9% |
| AU Consumer Goods | 0.0% | 0.0% |
| AU Consumer Services | 0.0% | 0.0% |
| AU Health Care | 3.7% | 8.5% |
| AU Industrials | 0.0% | 0.0% |
| AU Oil and Gas | 9.9% | 10.7% |
| AU REITS | 0.0% | 0.0% |
| AU Real Estate | 0.0% | 0.0% |
| AU Utilities | 1.0% | 3.9% |

As these portfolios were chosen on the basis of simulated outcomes, we can show the development of various quantities over time, including the best and worst case outcomes. In Figure 6 we show the percentiles of real assets over time for the 3.5% spending case, with the underlying data in Table 4. Analogously Figure 7 and Table 5 show the outcomes for the 4.5% case; for instance, we can see in Table 4 that in year 25 the 5% worst case outcome (i.e. the 5th percentile) for real assets is 2.5, which is 25% of the initial 10 that we used as an Acceptable Risk level.

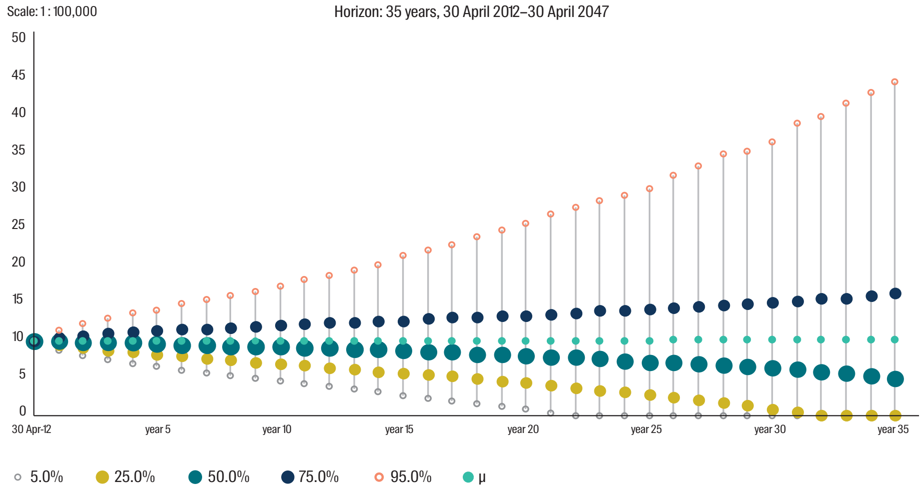
In both cases there is a steady divergence between the median (depicted as the large blue circle) and the mean (the smaller red circle). This indicates increasing skewness in the outcomes for real assets, and this is confirmed by the data in the tables, where the skewness (y-axis) and the kurtosis (x-axis) increase monotonously with time. On the upside, the best case outcomes become increasingly favourable with time. Table 5 also shows that in the 4.5% case there is at least a 5% probability the assets will be exhausted after 22 years.

## Figure 6: Real Assets Percentiles, CPI + 3.5%




Source: First Sentier Investors

## Figure 7: Real Assets Percentiles, CPI + 4.5%




Source: First Sentier Investors

Any forecasts in the charts/graphs represent hypothetical numbers and are purely for illustrative purposes only. The numbers do not represent actual or future performance.

Table 4

| Real Assets Percentiles, CPI + 3.5% | | | | | | DistributionStatistics | | | |
|---|---|---|---|---|---|---|---|---|---|
| Scale: 1:100,000 | 5% | 25% | 50% | 75% | 95% | μ | σ | γ | κ |
| 30/Apr/2012 | 10.0 | 10.0 | 10.0 | 10.0 | 10.0 | 10.0 | 0.0 | N/A | N/A |
| year 1 | 9.1 | 9.5 | 9.9 | 10.3 | 10.9 | 9.9 | 0.6 | 0.31 | 0.31 |
| year 2 | 8.6 | 9.3 | 9.8 | 10.5 | 11.4 | 9.9 | 0.9 | 0.42 | 0.50 |
| year 3 | 8.2 | 9.1 | 9.8 | 10.6 | 11.9 | 9.9 | 1.1 | 0.53 | 0.67 |
| year 4 | 7.9 | 8.9 | 9.8 | 10.7 | 12.2 | 9.9 | 1.4 | 0.61 | 1.02 |
| year 5 | 7.6 | 8.7 | 9.7 | 10.8 | 12.6 | 9.9 | 1.6 | 0.63 | 0.90 |
| year 6 | 7.2 | 8.6 | 9.6 | 10.9 | 13.0 | 9.8 | 1.8 | 0.69 | 0.88 |
| year 7 | 7.0 | 8.4 | 9.5 | 11.0 | 13.2 | 9.8 | 2.0 | 0.74 | 0.94 |
| year 8 | 6.8 | 8.3 | 9.5 | 11.1 | 13.6 | 9.8 | 2.1 | 0.74 | 0.82 |
| year 9 | 6.5 | 8.1 | 9.5 | 11.2 | 13.9 | 9.8 | 2.3 | 0.77 | 0.98 |
| year 10 | 6.3 | 7.9 | 9.4 | 11.3 | 14.3 | 9.8 | 2.5 | 0.83 | 1.11 |
| year 11 | 6.1 | 7.8 | 9.3 | 11.3 | 14.7 | 9.7 | 2.7 | 0.9 | 1.4 |
| year 12 | 5.7 | 7.6 | 9.3 | 11.5 | 15.0 | 9.7 | 2.9 | 0.9 | 1.4 |
| year 13 | 5.6 | 7.5 | 9.2 | 11.4 | 15.4 | 9.7 | 3.1 | 1.0 | 1.7 |
| year 14 | 5.3 | 7.3 | 9.2 | 11.5 | 15.8 | 9.7 | 3.3 | 1.0 | 1.6 |
| year 15 | 5.0 | 7.2 | 9.1 | 11.6 | 16.2 | 9.7 | 3.5 | 1.0 | 1.4 |
| year 16 | 4.8 | 7.1 | 9.0 | 11.7 | 16.4 | 9.6 | 3.7 | 1.0 | 1.5 |
| year 17 | 4.5 | 6.8 | 9.0 | 11.7 | 16.7 | 9.6 | 3.8 | 1.0 | 1.7 |
| year 18 | 4.3 | 6.7 | 9.0 | 11.7 | 17.0 | 9.6 | 4.0 | 1.0 | 1.6 |
| year 19 | 4.1 | 6.6 | 8.8 | 11.8 | 17.5 | 9.6 | 4.3 | 1.1 | 1.9 |
| year 20 | 3.9 | 6.4 | 8.7 | 11.9 | 17.8 | 9.6 | 4.5 | 1.2 | 2.3 |
| year 21 | 3.6 | 6.2 | 8.6 | 11.9 | 18.4 | 9.6 | 4.7 | 1.3 | 2.6 |
| year 22 | 3.4 | 6.0 | 8.6 | 12.0 | 18.9 | 9.5 | 4.9 | 1.3 | 2.8 |
| year 23 | 3.0 | 5.9 | 8.4 | 12.1 | 19.1 | 9.5 | 5.1 | 1.3 | 2.7 |
| year 24 | 2.8 | 5.7 | 8.4 | 12.1 | 19.6 | 9.5 | 5.4 | 1.3 | 2.7 |
| year 25 | 2.5 | 5.5 | 8.3 | 12.3 | 19.9 | 9.4 | 5.6 | 1.3 | 2.8 |
| year 26 | 2.3 | 5.3 | 8.1 | 12.3 | 20.4 | 9.4 | 5.9 | 1.3 | 2.9 |
| year 27 | 2.0 | 5.2 | 8.0 | 12.4 | 21.0 | 9.4 | 6.2 | 1.4 | 3.6 |
| year 28 | 1.7 | 5.0 | 8.0 | 12.4 | 21.6 | 9.4 | 6.5 | 1.5 | 4.2 |
| year 29 | 1.4 | 4.7 | 8.0 | 12.6 | 21.8 | 9.4 | 6.7 | 1.5 | 4.0 |
| year 30 | 1.1 | 4.5 | 7.9 | 12.5 | 22.6 | 9.3 | 7.0 | 1.5 | 3.8 |
| year 31 | 0.8 | 4.3 | 7.7 | 12.6 | 23.2 | 9.3 | 7.2 | 1.5 | 3.4 |
| year 32 | 0.5 | 4.1 | 7.6 | 12.8 | 23.9 | 9.3 | 7.5 | 1.5 | 3.9 |
| year 33 | 0.2 | 3.8 | 7.6 | 12.8 | 24.4 | 9.2 | 7.8 | 1.6 | 4.3 |
| year 34 | 0.0 | 3.6 | 7.5 | 13.0 | 24.7 | 9.2 | 8.1 | 1.6 | 4.4 |
| year 35 | 0.0 | 3.4 | 7.3 | 13.0 | 25.3 | 9.2 | 8.4 | 1.7 | 5.0 |

Table 5

| Real Assets Percentiles, CPI + 4.5% | | | | | | DistributionStatistics | | | |
|---|---|---|---|---|---|---|---|---|---|
| Scale: 1:100,000 | 5% | 25% | 50% | 75% | 95% | μ | σ | γ | κ |
| 30/Apr/2012 | 10.0 | 10.0 | 10.0 | 10.0 | 10.0 | 10.0 | 0.0 | N/A | N/A |
| year 1 | 8.7 | 9.3 | 9.9 | 10.4 | 11.3 | 9.9 | 0.8 | 0.36 | 0.25 |
| year 2 | 8.0 | 9.0 | 9.8 | 10.7 | 12.1 | 9.9 | 1.3 | 0.55 | 0.78 |
| year 3 | 7.4 | 8.7 | 9.8 | 10.9 | 12.9 | 9.9 | 1.7 | 0.67 | 1.04 |
| year 4 | 7.0 | 8.4 | 9.7 | 11.2 | 13.6 | 9.9 | 2.1 | 0.87 | 2.17 |
| year 5 | 6.7 | 8.2 | 9.6 | 11.4 | 14.0 | 9.9 | 2.4 | 0.91 | 2.05 |
| year 6 | 6.1 | 8.0 | 9.5 | 11.5 | 14.8 | 9.9 | 2.8 | 0.97 | 2.00 |
| year 7 | 5.8 | 7.7 | 9.3 | 11.6 | 15.4 | 9.8 | 3.1 | 1.01 | 2.01 |
| year 8 | 5.3 | 7.4 | 9.3 | 11.7 | 16.0 | 9.9 | 3.4 | 1.04 | 1.84 |
| year 9 | 5.0 | 7.2 | 9.2 | 11.9 | 16.4 | 9.9 | 3.7 | 1.13 | 2.39 |
| year 10 | 4.7 | 6.9 | 9.2 | 12.1 | 17.2 | 9.9 | 4.1 | 1.22 | 2.72 |
| year 11 | 4.3 | 6.7 | 9.0 | 12.2 | 18.0 | 9.9 | 4.5 | 1.3 | 3.5 |
| year 12 | 3.9 | 6.4 | 9.0 | 12.4 | 18.5 | 9.9 | 4.8 | 1.4 | 3.6 |
| year 13 | 3.5 | 6.3 | 8.9 | 12.4 | 19.3 | 9.9 | 5.2 | 1.5 | 3.9 |
| year 14 | 3.2 | 6.0 | 8.8 | 12.6 | 20.0 | 9.9 | 5.6 | 1.5 | 3.9 |
| year 15 | 2.8 | 5.8 | 8.6 | 12.6 | 21.1 | 9.9 | 5.9 | 1.5 | 3.4 |
| year 16 | 2.4 | 5.6 | 8.6 | 12.9 | 21.9 | 9.9 | 6.2 | 1.5 | 3.7 |
| year 17 | 2.0 | 5.3 | 8.5 | 13.0 | 22.6 | 9.9 | 6.6 | 1.5 | 3.9 |
| year 18 | 1.7 | 5.0 | 8.2 | 13.0 | 23.7 | 9.9 | 7.0 | 1.5 | 3.8 |
| year 19 | 1.3 | 4.7 | 8.2 | 13.2 | 24.6 | 9.9 | 7.5 | 1.7 | 4.2 |
| year 20 | 0.9 | 4.4 | 8.0 | 13.3 | 25.4 | 9.9 | 8.1 | 1.8 | 5.1 |
| year 21 | 0.4 | 4.1 | 7.8 | 13.5 | 26.7 | 10.0 | 8.6 | 2.0 | 7.1 |
| year 22 | 0.0 | 3.8 | 7.7 | 13.7 | 27.6 | 10.0 | 9.1 | 2.0 | 7.4 |
| year 23 | 0.0 | 3.5 | 7.6 | 14.0 | 28.4 | 10.0 | 9.6 | 2.0 | 7.3 |
| year 24 | 0.0 | 3.2 | 7.3 | 14.0 | 29.1 | 10.0 | 10.1 | 2.1 | 8.2 |
| year 25 | 0.0 | 2.9 | 7.2 | 14.2 | 30.1 | 10.0 | 10.7 | 2.2 | 9.1 |
| year 26 | 0.0 | 2.5 | 7.0 | 14.4 | 31.9 | 10.0 | 11.3 | 2.2 | 9.3 |
| year 27 | 0.0 | 2.1 | 6.9 | 14.5 | 33.0 | 10.1 | 12.1 | 2.4 | 10.7 |
| year 28 | 0.0 | 1.8 | 6.8 | 14.6 | 34.6 | 10.1 | 12.8 | 2.5 | 11.9 |
| year 29 | 0.0 | 1.4 | 6.5 | 14.8 | 35.0 | 10.0 | 13.3 | 2.3 | 9.2 |
| year 30 | 0.0 | 1.0 | 6.4 | 15.0 | 36.2 | 10.1 | 14.0 | 2.3 | 8.6 |
| year 31 | 0.0 | 0.6 | 6.2 | 15.2 | 38.7 | 10.1 | 14.7 | 2.2 | 8.1 |
| year 32 | 0.0 | 0.1 | 5.8 | 15.5 | 39.5 | 10.1 | 15.6 | 2.4 | 9.3 |
| year 33 | 0.0 | 0.0 | 5.6 | 15.6 | 41.4 | 10.1 | 16.4 | 2.5 | 10.4 |
| year 34 | 0.0 | 0.0 | 5.4 | 15.9 | 42.7 | 10.1 | 17.2 | 2.6 | 11.7 |
| year 35 | 0.0 | 0.0 | 5.1 | 16.2 | 44.1 | 10.1 | 18.3 | 2.8 | 15.1 |

Since we are interested in portfolios that outperform inflation by a certain amount, we can also look at the conditional outcomes for both portfolio returns as well as real asset developments over time. For the sake of consistency with the previous charts and graphs, we highlight in

Table 6 the bivariate probability distribution for inflation and GDP growth in the 25th year of the analysis. This table shows that in the 2,500 simulations, 9.4% had a realised inflation rate of more than 3.5% and simultaneously GDP growth of less than or equal to 2.5%. These probabilities form a base plane upon which we can project other data.

Table 6

| Bivariate Probability Distribution, CPI+3.5% | | | | | | |
|---|---|---|---|---|---|---|
| | Total | 20.3% | 29.4% | 29.3% | 21.0% | 100.0% |
| Price Inflation Australia, year25,scale: 1:0.01 | ... > 3.5% | 9.4% | 8.0% | 3.9% | 1.5% | 22.8% |
| | 2.5% < ... ≤ 3.5% | 5.4% | 8.3% | 8.3% | 3.5% | 25.6% |
| | 1.5% < ... ≤ 2.5% | 4.2% | 7.3% | 10.0% | 6.3% | 27.8% |
| | ... ≤ 1.5% | 1.3% | 5.8% | 7.1% | 9.7% | 23.8% |
| | | **... ≤ 2%** | **2% < ... ≤ 3%** | **3% < ... ≤ 4%** | **... > 4%** | **Total** |
| | | **GDP Australia, year 25, scale: 1:0.01** | | | | |

In Table 7 we can see that in these 9.4% of outcomes the nominal expected portfolio return is 5.94% for the CPI+4.5% case. For all periods of inflation higher than 3.5% the expected portfolio return is 7.60%, decreasing to 7.55%, 6.24% and finally ticking up to 6.50% with decreasing inflation rates. This is a favourable result showing that the portfolio does retain some inflation hedging characteristics while still meeting the desired return objective. We can also see in the table that the portfolio is strongly exposed to increasing economic growth; the expected return increases from 4.92% in periods of GDP growth below 2.5% to 9.11% in periods of GDP growth above 4.0%. Table 8 shows very similar outcomes for the CPI+3.5% case, albeit with lower nominal levels of return.

Table 7

| Expected Portfolio Return, year 25, CPI+4.5%, 30 Apr 2036–30 Apr 2037 Scale 1 : 0.01 | | | | | | |
|---|---|---|---|---|---|---|
| | Total | 4.92 | 5.55 | 8.20 | 9.11 | 6.95 |
| Price Inflation Australia, year25,scale: 1:0.01 | ... > 3.5% | 5.94 | 7.40 | 9.62 | 13.65 | 7.60 |
| | 2.5% < ... ≤ 3.5% | 4.77 | 5.97 | 9.36 | 11.30 | 7.55 |
| | 1.5% < ... ≤ 2.5% | 3.95 | 4.47 | 7.66 | 7.60 | 6.24 |
| | ... ≤ 1.5% | 1.25 | 3.75 | 6.82 | 8.59 | 6.50 |
| | | **... ≤ 2%** | **2% < ... ≤ 3%** | **3% < ... ≤ 4%** | **... > 4%** | **Total** |
| | | **GDP Australia, year 25, scale: 1:0.01** | | | | |

Table 8

| Expected Portfolio Return, year 25, CPI+3.5%, 30 Apr 2036–30 Apr 2037 Scale: 1:0.01 | | | | | | |
|---|---|---|---|---|---|---|
| | Total | 4.64 | 5.01 | 6.70 | 7.04 | 5.86 |
| Price Inflation Australia, year25, scale: 1:0.01 | ... > 3.5% | 5.46 | 6.23 | 7.73 | 10.36 | 6.45 |
| | 2.5% < ... ≤ 3.5% | 4.70 | 5.19 | 7.67 | 8.41 | 6.33 |
| | 1.5% < ... ≤ 2.5% | 3.64 | 4.41 | 6.21 | 6.02 | 5.30 |
| | ... ≤ 1.5% | 1.71 | 3.81 | 5.67 | 6.68 | 5.42 |
| | | **... ≤ 2%** | **2% < ... ≤ 3%** | **3% < ... ≤ 4%** | **... > 4%** | **Total** |
| | | **GDP Australia, year 25, scale: 1:0.01** | | | | |

Analogously we can show the level of expected real assets in year 25 of the simulation, which we have done in Table 9 for the CPI+4.5% case and in Table 10 for the CPI+3.5% case. The expected level of real assets in both cases is close to the initial level of 10, decreasing somewhat with slower GDP growth and with higher inflation. We see the highest level of expected real assets in both cases when inflation is low, which makes sense as this works in favour of real assets both in the indexation of spending as well as the assets themselves.

Table 9

| | | Expected Real Assets, year 25, CPI+4.5%, 30 Apr 2036–30 Apr 2037 Scale: 1:100000 | | | | |
|---|---|---|---|---|---|---|
| | Total | 9.65 | 9.48 | 10.0 | 10.96 | 9.98 |
| Price Inflation Australia, year 25, scale: 1:0.01 | ... > 3.5% | 9.78 | 8.89 | 11.0 | 8.9 | 9.62 |
| | 2.5% < ... ≤ 3.5% | 9.89 | 9.22 | 9.78 | 11.84 | 9.9 |
| | 1.5% < ... ≤ 2.5% | 9.0 | 9.51 | 9.36 | 10.69 | 9.64 |
| | ... ≤ 1.5% | 9.86 | 10.66 | 10.6 | 11.14 | 10.8 |
| | | ... ≤ 2% | 2% < ... ≤ 3% | 3% < ... ≤ 4% | ... > 4% | Total |
| | | GDP Australia, year 25, scale: 1:0.01 | | | | |

Table 10

| | | Expected Real Assets, year 25, CPI+3.5%, 30 Apr 2036–30 Apr 2037 Scale: 1:100000 | | | | |
|---|---|---|---|---|---|---|
| | Total | 9.21 | 9.17 | 9.47 | 10.03 | 9.45 |
| Price Inflation Australia, year 25, scale: 1:0.01 | ... > 3.5% | 9.29 | 8.83 | 9.82 | 8.98 | 9.2 |
| | 2.5% < ... ≤ 3.5% | 9.3 | 9.01 | 9.52 | 10.39 | 9.43 |
| | 1.5% < ... ≤ 2.5% | 8.91 | 9.27 | 9.12 | 9.99 | 9.32 |
| | ... ≤ 1.5% | 9.3 | 9.76 | 9.7 | 10.1 | 9.86 |
| | | ... ≤ 2% | 2% < ... ≤ 3% | 3% < ... ≤ 4% | ... > 4% | Total |
| | | GDP Australia, year 25, scale: 1:0.01 | | | | |

Other cross-sections can be fruitful to explore and analyse in addition to the inflation/GDP case described here, but in the interest of brevity we have omitted further such analyses from this paper. Overall though the portfolios thusly designed and selected seem to meet the required criteria, and behave more or less in the way one would expect. Naturally this depends crucially on the validity and accuracy of the modelling that we employed. Checking the actual historical performance against GDP and inflation does corroborate in broad terms the forward-looking simulated results.

## Active Management: Adding tracking error and alpha

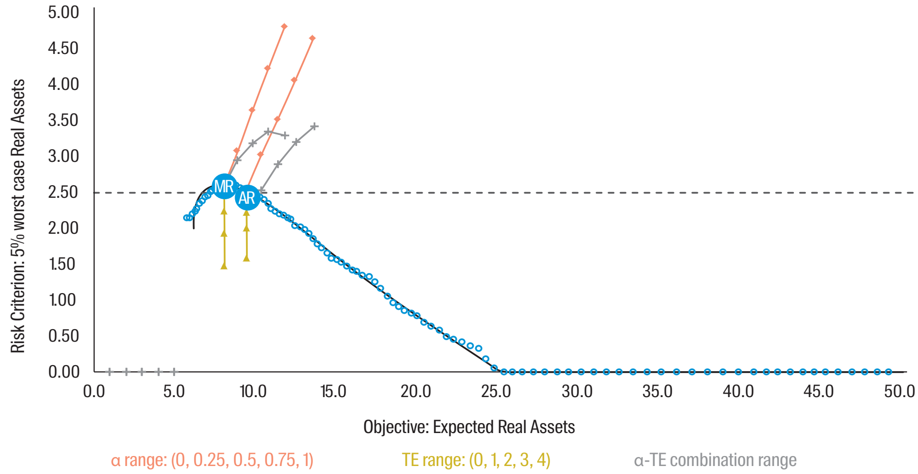
Having established optimal strategies, the next step is one of active implementation. We have used only pure β exposure up until this point, which in most cases can be replicated passively. In many instances some form of active implementation does make sense though, and it is crucially important to assess the viable leeway for active management in the same risk/return space as the analysis. Setting a risk budget (or even a hedging budget) is a natural extension of setting a strategy,and a comprehensive methodology is described in *Baars, Kocourek and vander Lende, Integrated Approach to ALM, Risk Budgeting and Alpha 2012.*

Taking the Acceptable Risk portfolios for both cases as the starting point, we perturb the allocation in three ways:

1. By adding tracking error with zero alpha to the portfolio
2. By adding uncorrelated alpha with zero tracking error to the portfolio
3. By adding a combination of uncorrelated alpha and tracking error with a fixed information ratio to the portfolio

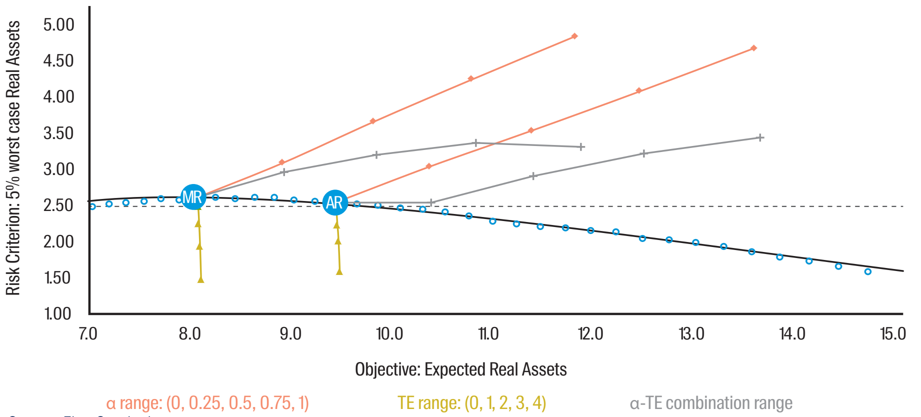
For each perturbation we calculate the resulting objective function (expected real assets) and risk criterion (worst-case real assets). The results for the CPI+3.5% case are plotted in Figure 8, with a zoomed-in version in Figure 9. The CPI+4.5% case is shown in Figure 10 with the zoomed-in version in Figure 11.

Figure 8: Investment Strategy Risk Analysis, CPI + 3.5%



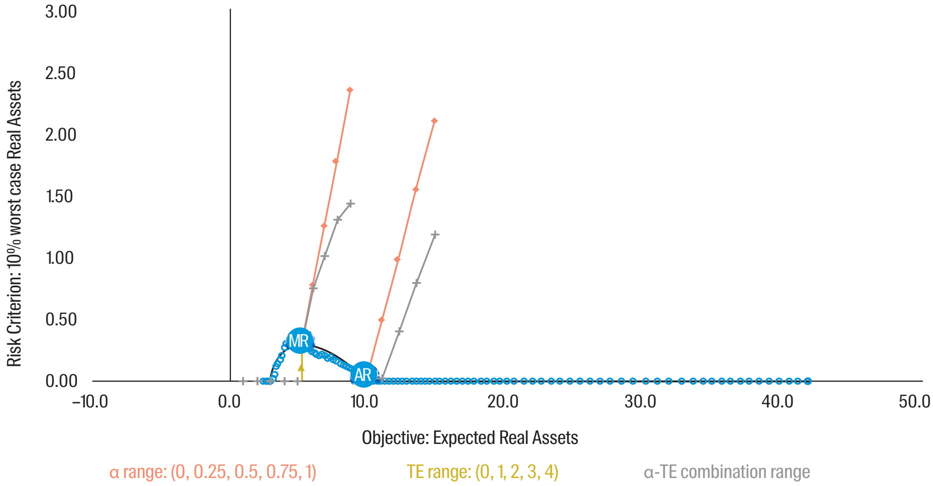

Source: First Sentier Investors

Figure 9: Investment Strategy Risk Analysis, CPI + 3.5%



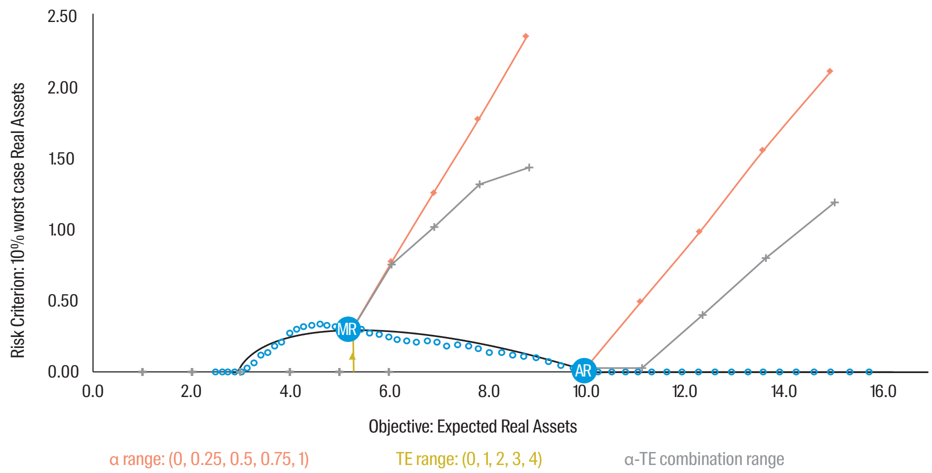

Source: First Sentier Investors

Figure 10: Investment Strategy Risk Analysis, CPI + 4.5%




Source: First Sentier Investors

Figure 11: Investment Strategy Risk Analysis, CPI + 4.5%




Source: First Sentier Investors

These charts show that adding tracking error with no α results in a deterioration in the worst-case outcome, but has no effect on the expected outcome. This is also what one would expect as it is merely adding volatility without changing the mean of a distribution. This holds true in both the CPI+3.5% as well as the CPI+4.5% case. In all four figures this is represented by the green lines with triangular markers.[7]

Adding uncorrelated α improves both the expected outcome as well as the worst-case outcome. After all, we are adding "pure performance" to the portfolios without any penalty. It is interesting to note the difference between the two cases though: the slope of the α line (shown in purple with diamond markers) is much steeper for the CPI+4.5% case than for the CPI+3.5% case. The improvement in expected real assets is stronger in the former case, increasing from 9.98 to 14.94, whereas in the latter case the improvement is smaller, from 9.45 to 13.62. The outcomes are also tabulated in Table 11.

[7] Note that for the CPI+4.5% case adding tracking error only leads to exhaustion of assets for tracking errors of more than 2% or so.
Any forecasts in the charts/graphs represent hypothetical numbers and are purely for illustrative purposes only. The numbers do not represent actual or future performance.

Table 11

| | | Adding α and Tracking Error | | | |
|---|---|---|---|---|---|
| | | CPI+3.5% | | CPI+4.5% | |
| | | Expected Real Assets | 5% Worst Case Real Assets | Expected Real Assets | 10% Worst Case Real Assets |
| Basecase | | 9.45 | 2.53 | 9.98 | 0.03 |
| α Overlay | 0.00% | 9.45 | 2.53 | 9.98 | 0.03 |
| | 0.25% | 10.39 | 3.03 | 11.10 | 0.49 |
| | 0.50% | 11.40 | 3.52 | 12.30 | 0.99 |
| | 0.75% | 12.48 | 4.06 | 13.58 | 1.56 |
| | 1.00% | 13.62 | 4.65 | 14.94 | 2.11 |
| Tracking Error Overlay | 0.0% | 9.45 | 2.53 | 9.98 | 0.03 |
| | 1.0% | 9.46 | 2.44 | 10.00 | −0.02 |
| | 2.0% | 9.47 | 2.23 | 10.01 | −0.11 |
| | 3.0% | 9.48 | 2.01 | 10.03 | −0.25 |
| | 4.0% | 9.49 | 1.58 | 10.05 | −0.52 |
| α- Tracking Error Combination | | 9.45 | 2.53 | 9.98 | 0.03 |
| | 25.0% | 10.40 | 2.90 | 11.12 | 0.40 |
| | 25.0% | 11.43 | 3.21 | 12.34 | 0.80 |
| | 25.0% | 12.52 | 3.43 | 13.64 | 1.19 |
| | 25.0% | 13.68 | 3.45 | 15.04 | 1.38 |

Looking at the combination of adding both α and tracking error with a fixed information ratio of 0.25 we actually observe a flattening of the gray lines with the plus-shaped markers for the CPI+3.5% case. This means that the worst-case outcome ceases to improve once the tracking error exceeds about 2%, and the incremental benefit of adding tracking error (assuming constant information ratio) goes to zero as far as the worst case is concerned.

The expected outcome still shows improvement. We also see a flattening of the gray lines for the CPI+4.5% case, but it is much more gradual and the incremental benefit for the worst-case outcome still argues in favour of adding more tracking error at constant information ratio.

The decision on how much tracking error to allow depends on the investor's assessment of how much information ratio is feasible. This depends inter alia on the efficiency of the selected asset categories, availability of experienced managers, transaction costs, market impact and the feasibility of dynamic and tactical reallocations between the asset categories.

## Summary

The construction of "Inflation Plus" portfolios requires attention be paid to a number of aspects that play no role in a pure asset-only construction. The correlation with inflation itself, but also the real performance potential of asset categories are important. In the modeled case of Australia we have found it feasible to model well-behaved portfolios with investment objectives of CPI+3.5% and CPI+4.5%. In the expected case both can run more or less as perpetuities, but asset exhaustion is a risk in both cases on long horizons.

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